Rotational Equilibrium
and
Rotational Dynamics

Lets Talk Torque…

So Far…

- Up till now we have only dealt with Static Equilibrium. Also known as translational equilibrium, no movement.
- Conditions for equilibrium are

1. Object is motionless
2. Object is moving at a constant speed
3. Object is rotating at a constant speed

Torque

- Consider force required to open door. Is it easier to open the door by pushing/pulling away from hinge or close to hinge?

Farther from from hinge, larger rotational effect!

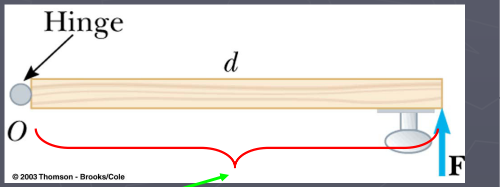
Physics concept: torque

Torque

- Torque, τ, is the tendency of a force to rotate an object about some axis

Door example:

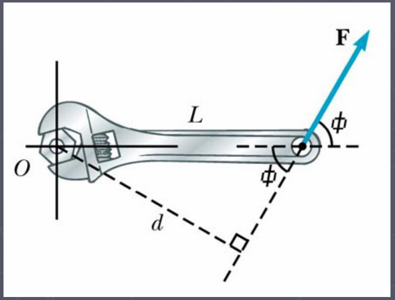
$$\boxed{\tau = Fd}$$



- τ is the torque
- d is the *lever arm* (or moment arm)
- F is the force

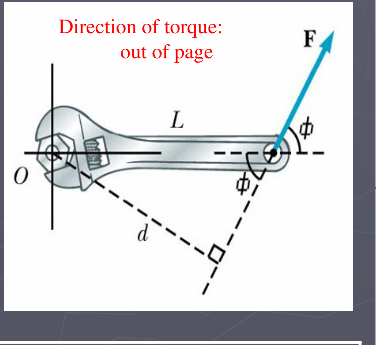
Lever Arm

- The lever arm, d, is the shortest *(perpendicular)* distance from the axis of rotation to a line drawn along the the direction of the force
  - d = L sin Φ
- It is not necessarily the distance between the axis of rotation and point where the force is applied



Direction of Torque

- Torque is a vector quantity
  - The direction is perpendicular to the plane determined by the lever arm and the force
  - Direction and sign:
    - If the turning tendency of the force is counterclockwise, the torque will be positive
    - If the turning tendency is clockwise, the torque will be negative



Units	
SI	Newton meter (Nm)
US Customary	Foot pound (ft lb)

An Alternative Look at Torque

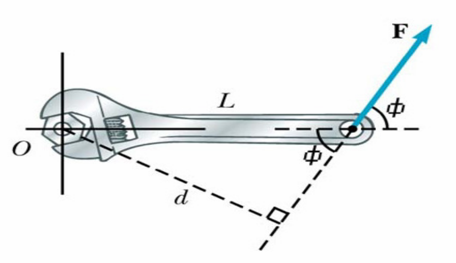
- The force could also be resolved into its x- and y-components
  - The x-component, F cos Φ, produces 0 torque
  - The y-component, F sin Φ, produces a non-zero torque

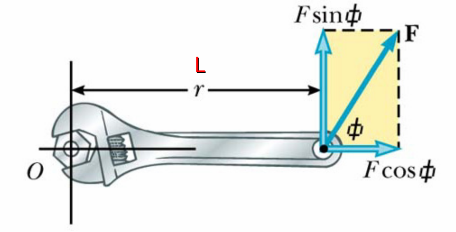
$$\boxed{\tau = FL\sin\phi}$$

F is the force

L is the distance along the object

Φ is the angle between force and object





ConceptTest 1

You are trying to open a door that is stuck by pulling on the doorknob in a direction perpendicular to the door. If you instead tie a rope to the doorknob and then pull with the same force, is the torque you exert increased? Will it be easier to open the door?

1. No
2. Yes

ConcepTest 1

Convince your neighbor!

You are trying to open a door that is stuck by pulling on the doorknob in a direction perpendicular to the door. If you instead tie a rope to the doorknob and then pull with the same force, is the torque you exert increased? Will it be easier to open the door?

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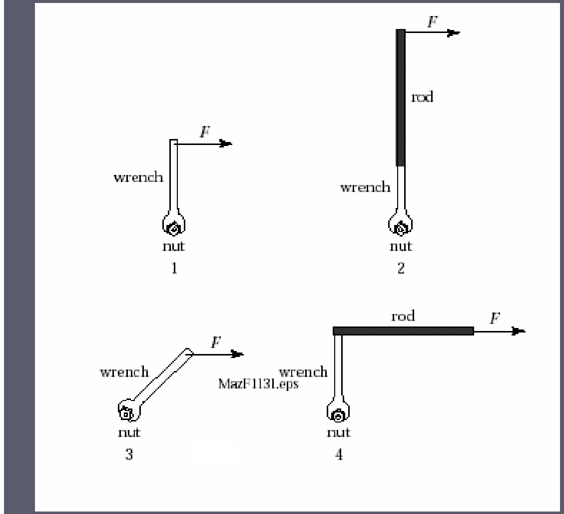
[✓ 1. No]
2. Yes

ConcepTest 2



You are using a wrench and trying to loosen a rusty nut. Which of the arrangements shown is most effective in loosening the nut? List in order of descending efficiency the following arrangements:

ConcepTest 2

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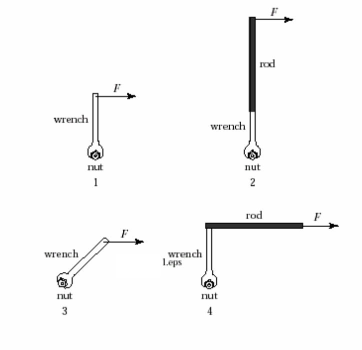
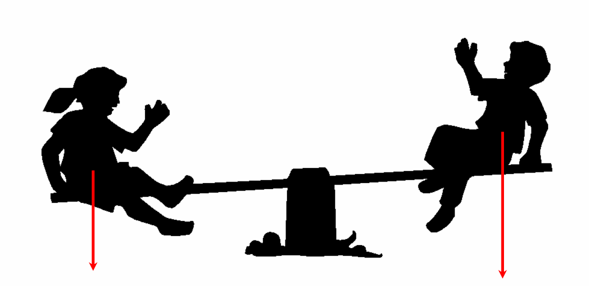


2, 1, 4, 3
or
2, 4, 1, 3

What if two or more different forces act on lever arm?

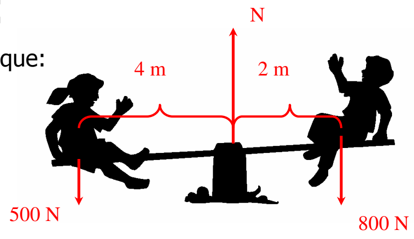


Net Torque

- The net torque is the sum of all the torques produced by all the forces
  - Remember to account for the direction of the tendency for rotation
    - Counterclockwise torques are positive
    - Clockwise torques are negative

Example 1:

Determine the net torque:



Given:

weights: w_1= 500 N
w_2 = 800 N
lever arms: d_1=4 m
d_2=2 m

Find:

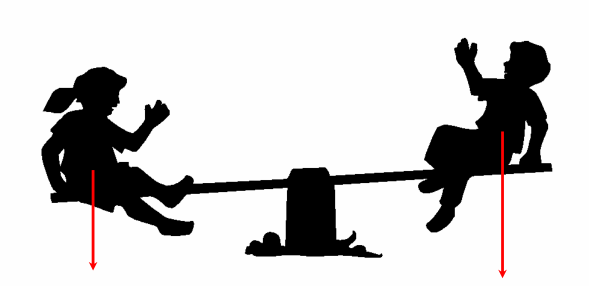
Στ = ?

1. Draw all applicable forces
2. Consider CCW rotation to be positive

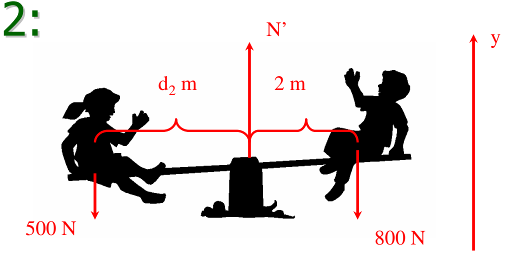
$$\begin{aligned}\sum \tau &= (500\,N)(4\,m) + (-)(800\,N)(2\,m)\\ &= +2000\,N\cdot m - 1600\,N\cdot m\\ &= +400\,N\cdot m \quad ✓\end{aligned}$$

Rotation would be CCW

Where would the 500 N person have to be relative to fulcrum for zero torque?



Example 2:



Given:

weights: $w_1 = 500\ N$
$w_2 = 800$ N
lever arms: $d_1 = 4\ m$
$\Sigma\tau = 0$

Find:

$d_2 = ?$

1. Draw all applicable forces and moment arms

$$\sum \tau_{RHS} = -(800\,N)(2\,m)$$
$$\sum \tau_{LHS} = (500\,N)(d_2\,m)$$
$$-800 \cdot 2\,[N \cdot m] + 500 \cdot d_2\,[N \cdot m] = 0 \quad \Rightarrow \quad d_2 = 3.2\,m \ ✓$$

According to our understanding of torque there would be no rotation and no motion!

↑

What does it say about acceleration and force?

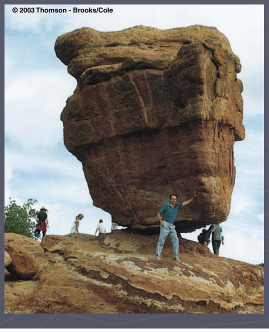
$$\sum F_i = (-500\,N) + N' + (-800\,N) = 0$$
$$N' = 1300\,N$$

Thus, according to 2nd Newton's law ΣF=0 and a=0!

Torque and Equilibrium

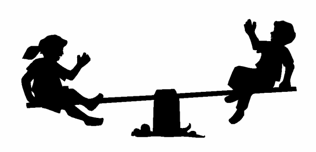
- First Condition of Equilibrium
  - The net external force must be zero

$$\boxed{\begin{array}{c}\Sigma \vec{F} = 0 \\ \Sigma F_x = 0 \text{ and } \Sigma F_y = 0\end{array}}$$



  - This is a necessary, but not sufficient, condition to ensure that an object is in complete mechanical equilibrium
  - This is a statement of translational equilibrium
- Second Condition of Equilibrium
  - The net external torque must be zero

$$\boxed{\Sigma \tau = 0}$$



  - This is a statement of rotational equilibrium

Axis of Rotation

- So far we have chosen obvious axis of rotation
- If the object is in equilibrium, it does not matter where you put the axis of rotation for calculating the net torque
  - The location of the axis of rotation is completely arbitrary
  - Often the nature of the problem will suggest a convenient location for the axis
  - When solving a problem, you *must* specify an axis of rotation
    - Once you have chosen an axis, you must maintain that choice consistently throughout the problem

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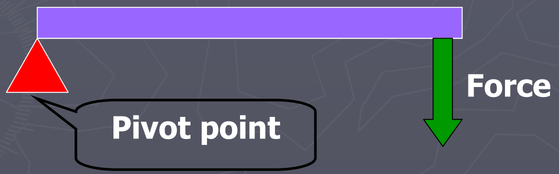
Axis of Rotation

- The axis of rotation is often also known as a pivot point.
- It may be represented as a small triangle…



Its often also known as the "**moment**"

Center of Gravity

- The force of gravity acting on an object must be considered
- In finding the torque produced by the force of gravity, all of the weight of the object can be considered to be concentrated at one point

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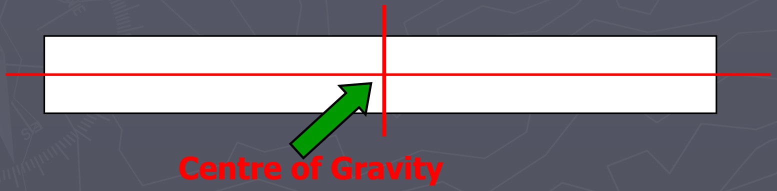
Centre of Gravity

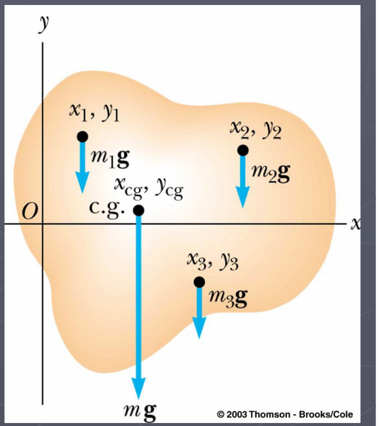
- Most of the situations we will encounter will allow us to find the centre easily. Look for the word "uniform", Then we can use the centre of symmetry.



Calculating the Center of Gravity

1. The object is divided up into a large number of very small particles of weight (mg)
2. Each particle will have a set of coordinates indicating its location (x,y)
3. The torque produced by each particle about the axis of rotation is equal to its weight times its lever arm
4. We wish to locate the point of application of the *single force* , whose magnitude is equal to the weight of the object, and whose effect on the rotation is the same as all the individual particles.

- This point is called the *center of gravity* of the object



Coordinates of the Center of Gravity

- The coordinates of the center of gravity can be found from the sum of the torques acting on the individual particles being set equal to the torque produced by the weight of the object

$$x_{cg} = \frac{\Sigma m_i x_i}{\Sigma m_i} \quad and \quad y_{cg} = \frac{\Sigma m_i y_i}{\Sigma m_i}$$

- The center of gravity of a homogenous, symmetric body must lie on the axis of symmetry.
- Often, the center of gravity of such an object is the *geometric* center of the object.

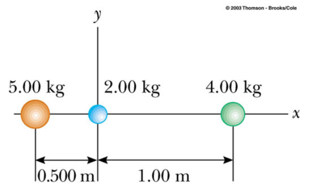
Example: Find center of gravity of the following system:

Given:

masses: $m_1 = 5.00$ kg
$m_2 = 2.00$ kg
$m_3 = 4.00$ kg
lever arms: $d_1 = 0.500\ m$
$d_2 = 1.00\ m$

Find:

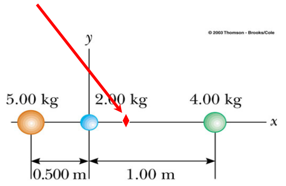
Center of gravity



$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{5.00\,kg(-0.500\,m) + 2.00\,kg(0\,m) + 4.00\,kg(1.00\,m)}{11.0\,kg}$$

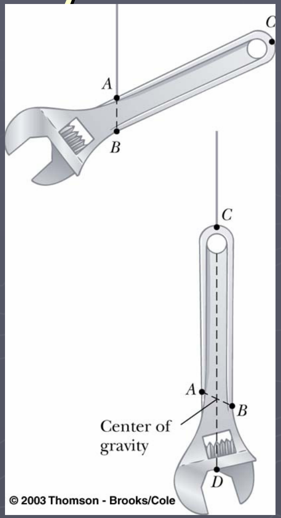
$$= 0.136\,m$$



✓

Experimentally Determining the Center of Gravity

- The wrench is hung freely from two different pivots
- The intersection of the lines indicates the center of gravity
- A rigid object can be balanced by a single force equal in magnitude to its weight as long as the force is acting upward through the object's center of gravity



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Equilibrium, once again

- A zero net torque does not mean the absence of rotational motion
  - An object that rotates at uniform angular velocity can be under the influence of a zero net torque
    - This is analogous to the translational situation where a zero net force does not mean the object is not in motion

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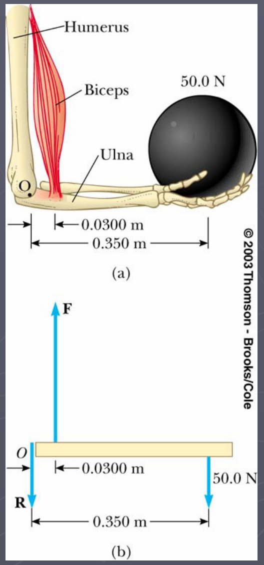
Example of a Free Body Diagram

- Isolate the object to be analyzed
- Draw the free body diagram for that object
  - Include all the external forces acting on the object



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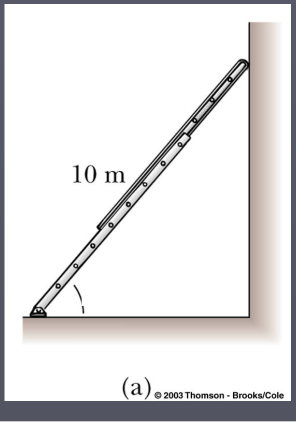
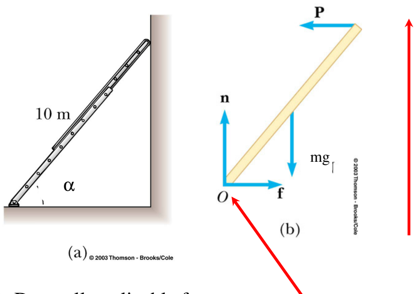
Example

Suppose that you placed a 10 m ladder (which weights 100 N) against the wall at the angle of 30°. What are the forces acting on it and when would it be in equilibrium?



Example:



Given:

weights: $w_1 = 100\ N$
length: $l = 10\ m$
angle: $\alpha = 30°$
$\Sigma\tau = 0$

Find:

$f = ?$
n=?
P=?

1. Draw all applicable forces
2. Choose axis of rotation at bottom corner (τ of *f* and *n* are 0!)

Torques:

$$\sum \tau = mg\frac{L}{2}\cos 30^\circ - PL\sin 30^\circ = 0$$

$$0 = 100\,N \cdot \frac{1}{2} \cdot 0.866 - P \cdot 1 \cdot \frac{1}{2}$$

$$P = 86.6\,N \quad ✓$$

Forces:

$$\sum F_x = f - P = 0$$

$$f = 86.6\,N$$

$$\sum F_y = n - mg = 0$$

$$n = 100\,N \quad ✓$$

Note: f = μ_s n, so $\mu_s = \frac{f}{n} = \frac{86.6\,N}{100\,N} = 0.866$

Your Turn…

- Text Reference: P230, Chapter 9-3, see example problems with worked solutions.
- Do the following from Page 250 Giancoli: Q #1, 3, 5, 7,10, 11, 21, 22, 23, 25, 26.

Torque Summary

- What is Torque?
- What is Rotational Equilibrium?
- Translational Equilibrium?
- What is a lever arm? Do we need to us this?
- Does it matter where the pivot point is?
- Do we need to satisfy all conditions for equilibrium?
- How do we figure directions out?