Gravitation

A strangely attractive topic

Objectives: by the end you will be able to…

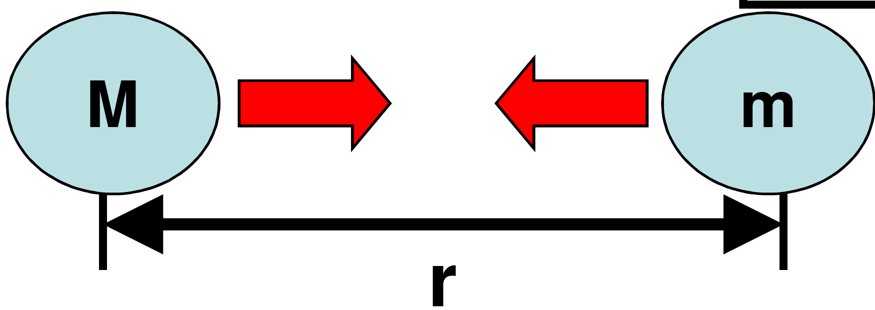
- State and apply Newtons law of Universal Gravitation
- Describe the Gravitational field of a body in terms of an inverse square relationship
- Describe the work required to move a n object in a gravitational field using a forve vs diatnce graph
- Calculate the work required to move an object in a gravitational field
- Solve problems involving
    - Grav. Potential energy relative to zero at infinity
    - Mass
    - Distance

Objectives: by the end you will be able to…

- Analyse and describe orbiting systems in terms of universal gravitation and centripetal forces
- Solve problems involving orbiting systems
- Calculate the total energy of an orbiting object

Newton's law of universal gravitation states that any two objects of masses m_1 and m_2 separated by a distance r will exert a gravitational force on each other.

Attractive !



$$F = G\frac{m_1 m_2}{d^2},\ G = 6.67 \times 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}}$$

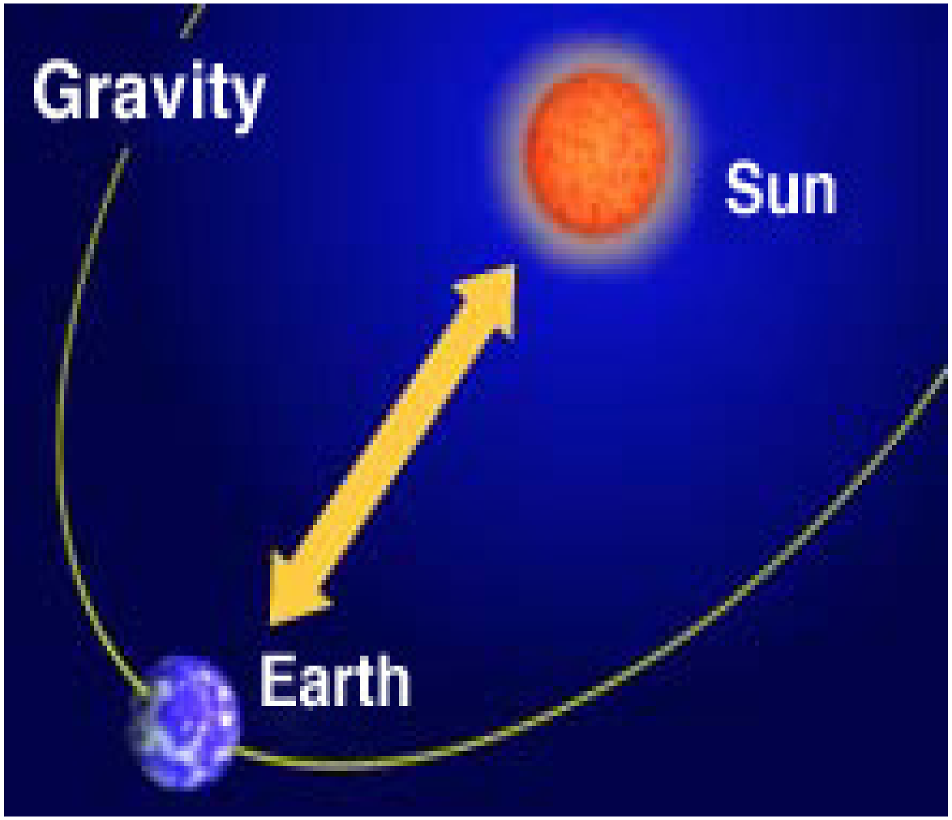
This gravitational force is an attractive force and is directly proportional to the product of the masses (F = m1m2) and inversely proportional to r^2 ($F = 1/r^2$).

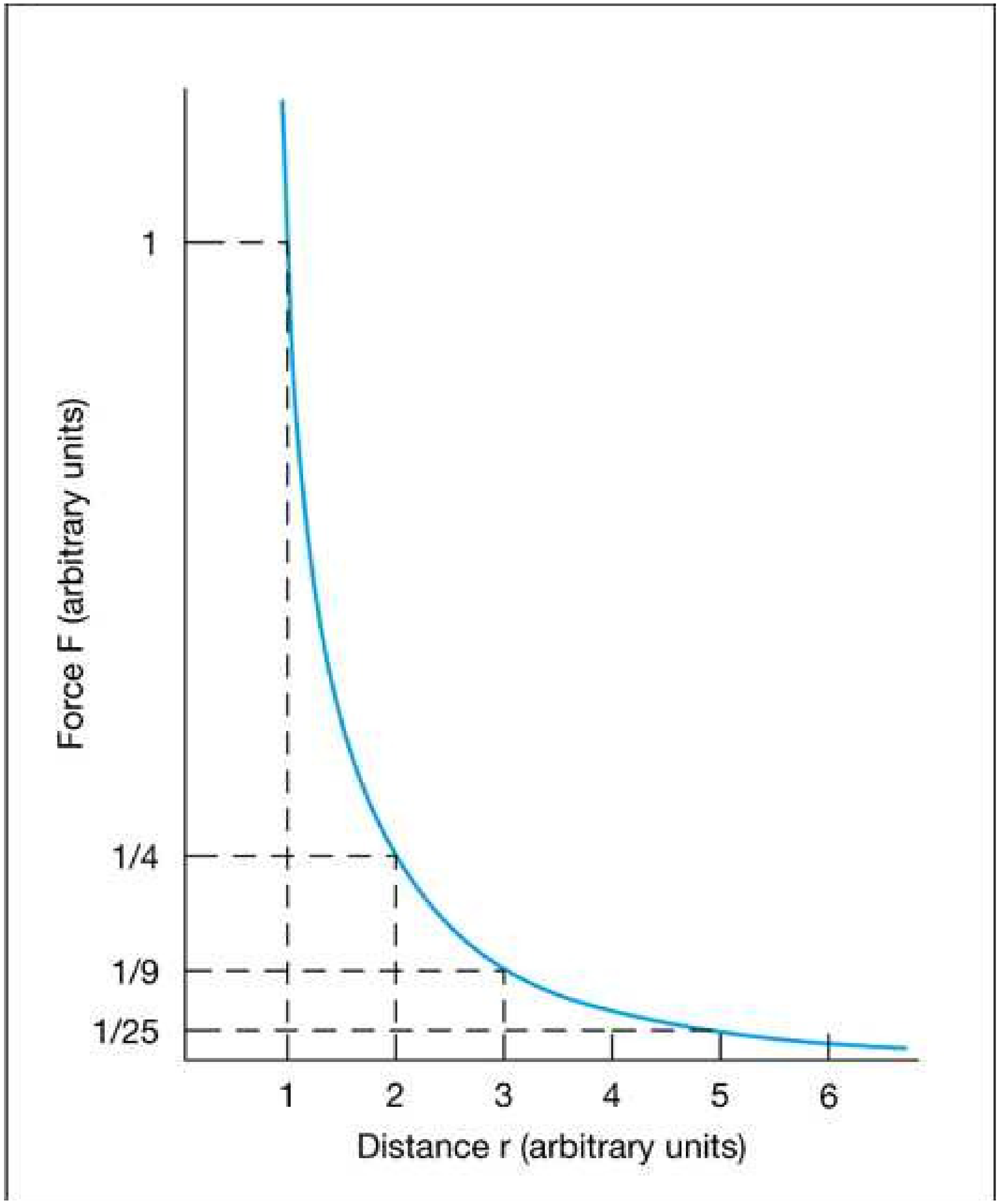
Nature has **FOUR** fundamental forces…

1. Gravity

- Only attractive.
- Only depends upon mass and separation.
- Weakest force but operates over any distance.

•Gravity operates over *any* range, and affects *anything* with mass.





$$F_g = G\frac{M_1M_2}{d^2}$$

Depends on the *inverse* of distance squared (d X d)!

"Inverse Square Law"

Bonding with the gravitational force equation…

Depends on the mass of both objects. Masses on the top implies the force goes up when M goes up.

$$F_g = G\frac{M_1 M_2}{d^2}$$

Amount of force

"Newton's Constant": sets how strong the force is. If it were different, gravity would be stronger or weaker.

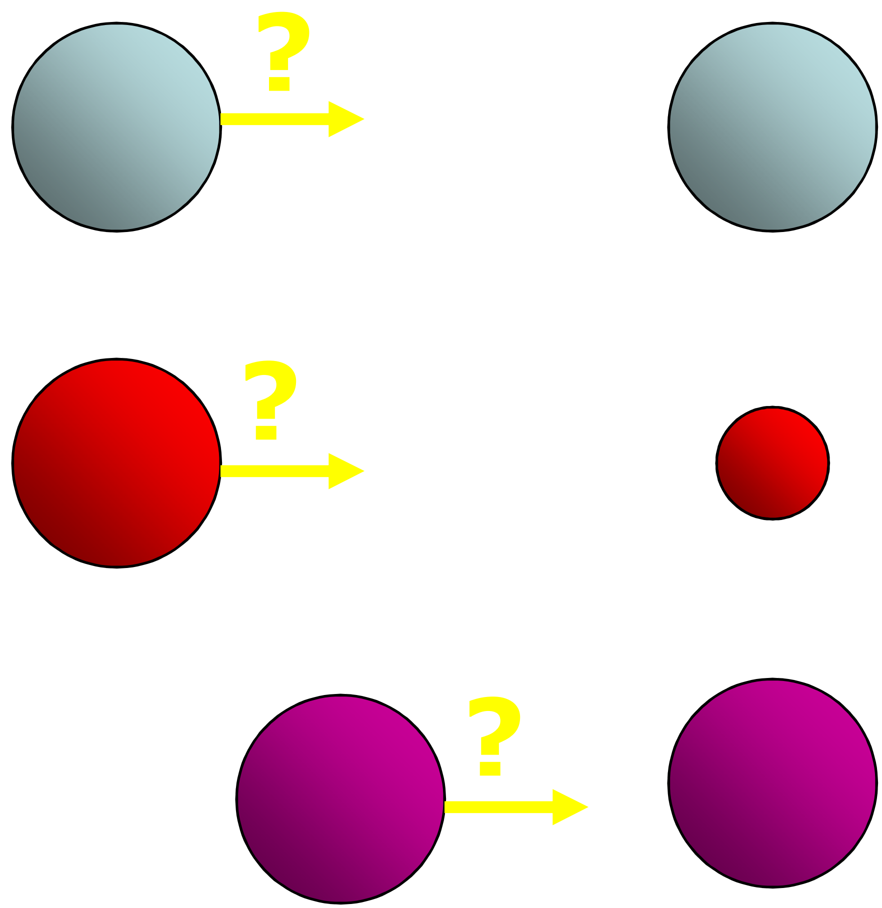
Depends on distance squared (d X d)! Much stronger dependence than on mass of either object. On bottom, so force is weaker when distance is bigger.

Which of the following is the weakest force? The strongest?

A (green).

B (red).

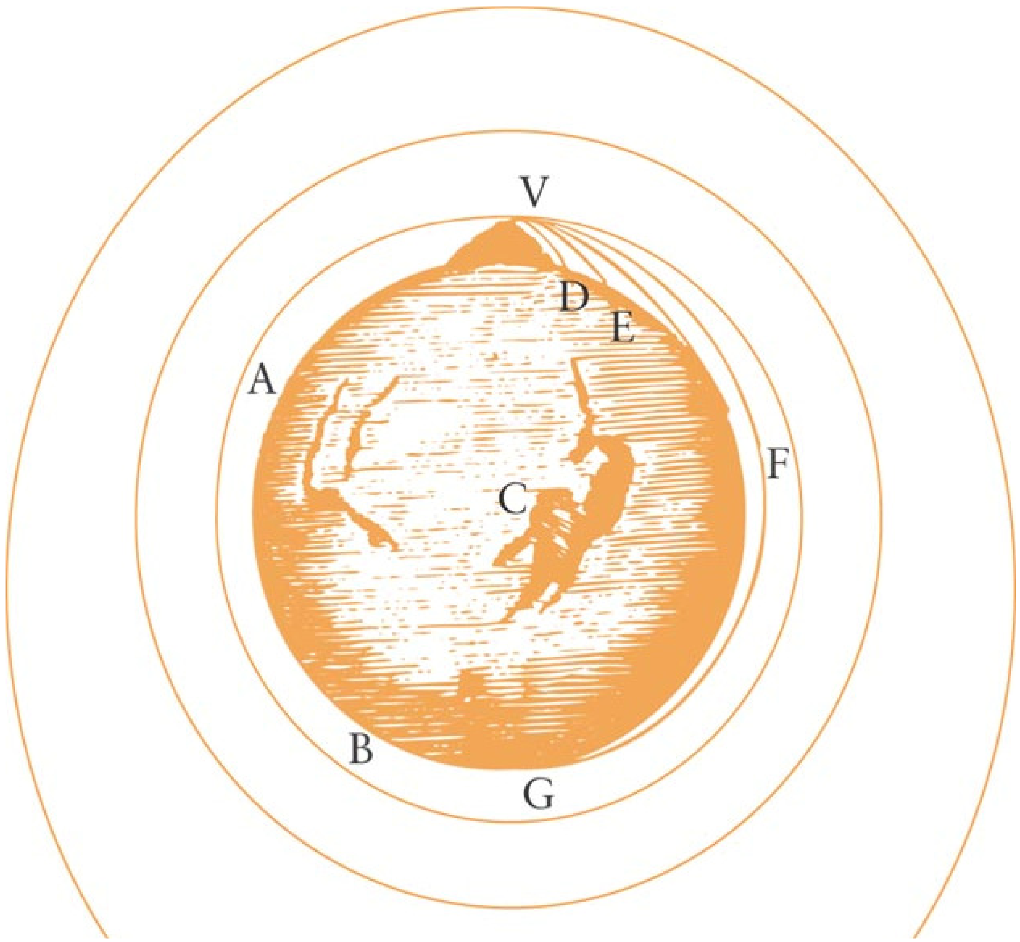
C (purple).



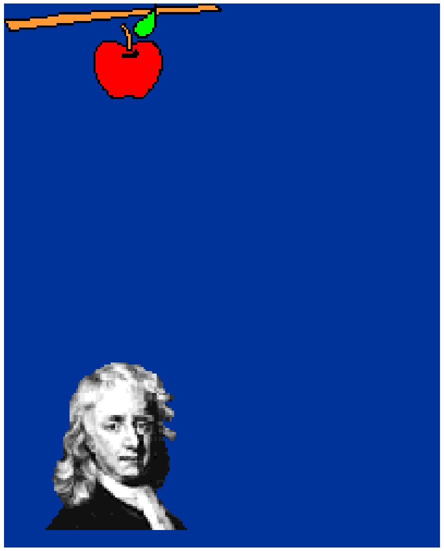
PLANETARY ORBITS

Newton's "Thought Experiment":

Throw a ball from a mountaintop faster and faster (neglecting air resistance). What happens ?



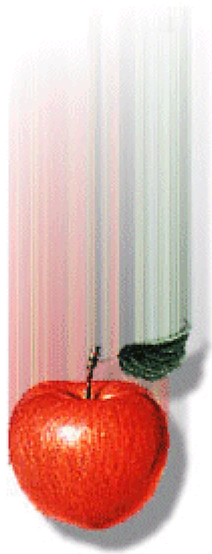
Newton and Apples?



There is a popular story that Newton was sitting under an apple tree, an apple fell on his head, and he suddenly thought of the Universal Law of Gravitation.

Most likely not true but what it may have done is cause him to understand that gravity acts on an apple no matter how high the tree!

Hence the other thought experiment.



The weight of an apple would be given as
w = mg

Where its weight is actually the force exerted on it by the Earth!

So it could also be

$$F_g = mg$$

Or

$$F_g = \frac{GMm}{d^2}$$

So if $F_g = F_g$

Then $\dfrac{GMm}{r^2} = mg$

$\therefore \mathrm{g} = \dfrac{\mathrm{GM}}{\mathrm{r}^2}$

Where:

M = planet mass

m = mass of the ‘apple’

g = acceleration due to gravity

r = distance between the objects centres

Let’s calculate g at 400 km altitude:

$W = m\,g = G\,M_E\,m/R^2$

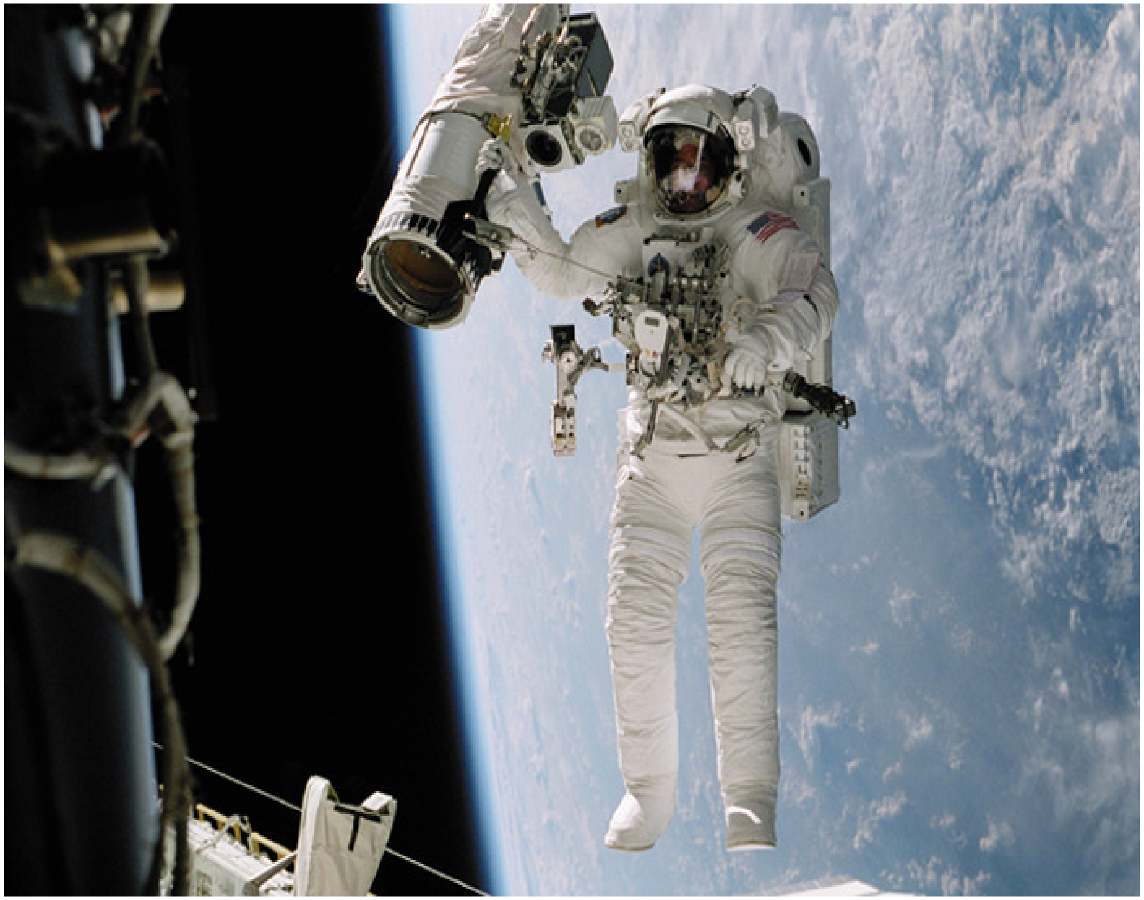
$g = G\,M_E/(R_E + 400\text{ km})^2 = 8.68\text{ m/s}^2$

Compare to 9.81 m/s^2 on the surface of the earth.

Astronauts in the space shuttle can hardly be called “weightless”.

It just seems that way because they are in continuous “free fall” as the orbit the earth.

If gravity is the only force, the acceleration of an object *doesn't* depend on the object's mass



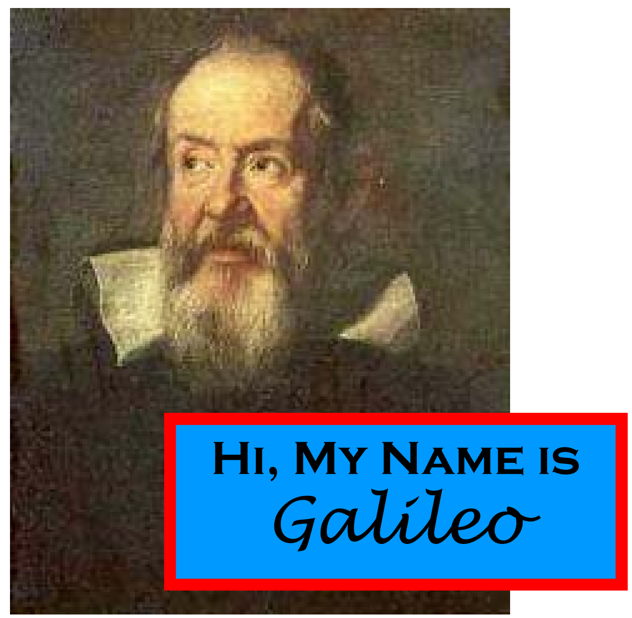
Motion depends on every *other* mass in the universe, but *not your own*!

If gravitational acceleration is independent of the mass of the moving body, all objects on Earth accelerate towards the ground at the same rate.

The classic Tower of Pisa drop

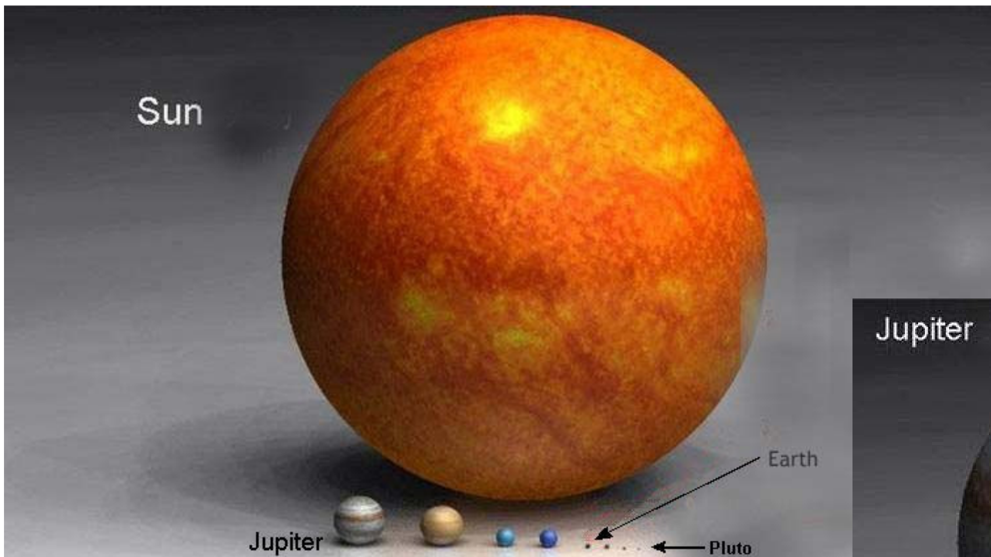
- Iron & Bronze spheres fell at the same rate

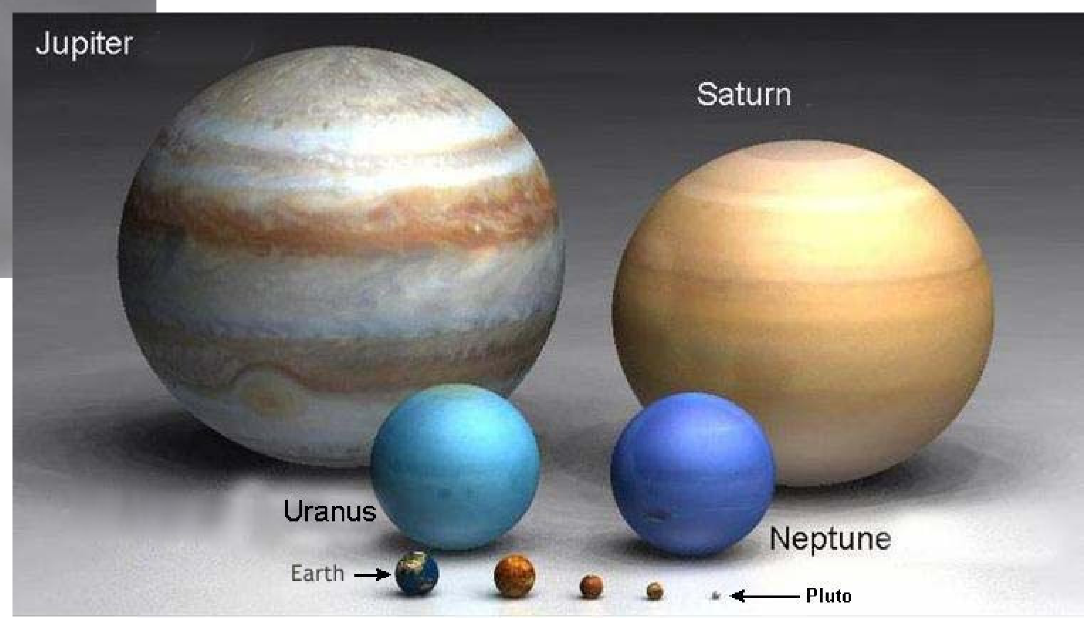
SPLAT!



The Sun is about 300,000 times more massive than the Earth, and about 1000 times more massive than the largest planet

it is a good initial approximation to neglect all interactions except that of the planet and the Sun.

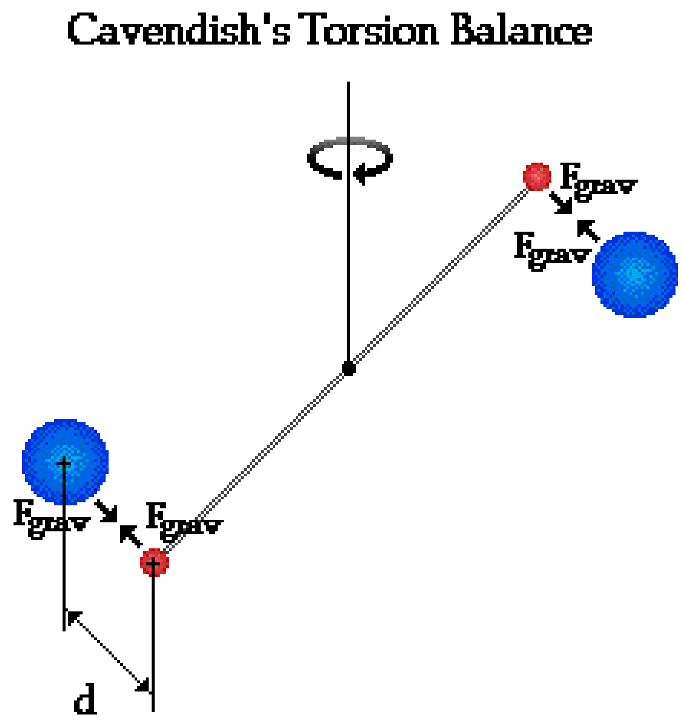




The Cavendish Experiment

"Measuring the Mass of the Earth" - 1798

Henry Cavendish (1731-1810) developed an apparatus for experimentally determining the value of **G** involved a light, rigid rod which was 6-feet long. Two small lead spheres were attached to the ends of the rod and the rod was suspended by a wire. The angle of rotation yields the amount of torsional force. A diagram of the apparatus is shown below:



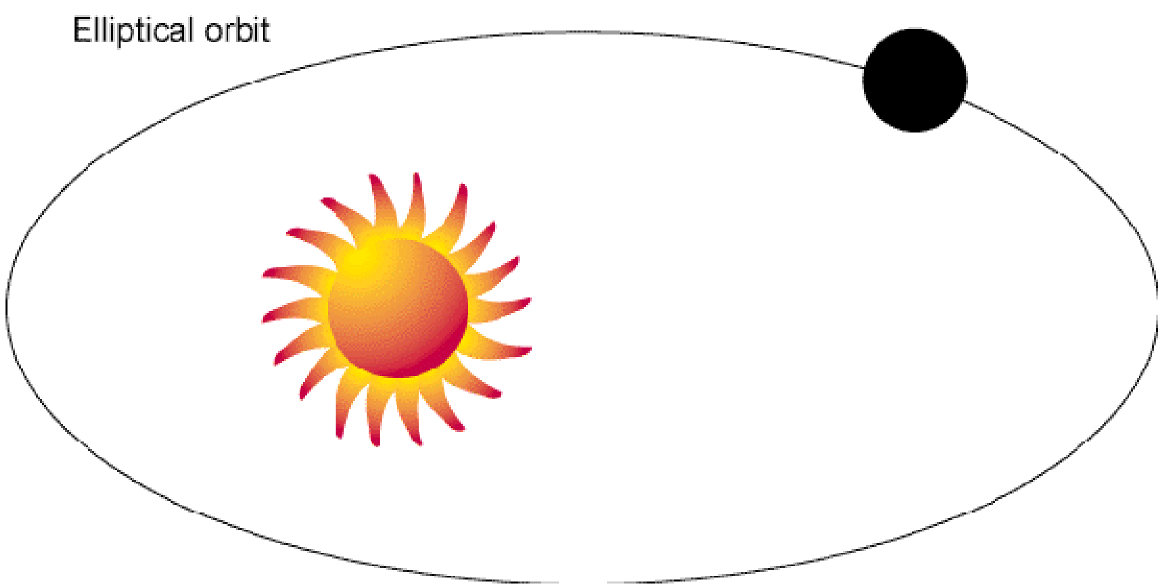
The Laws of Planetary Motion

- Kepler studied the observations of planetary motion and found that he could explain them with a much simpler model
- His model is expressed as three laws
  1. Law of Ellipses
  2. Law of Equal Areas
  3. Harmonic Law

Tycho Brahe (1546-1601)

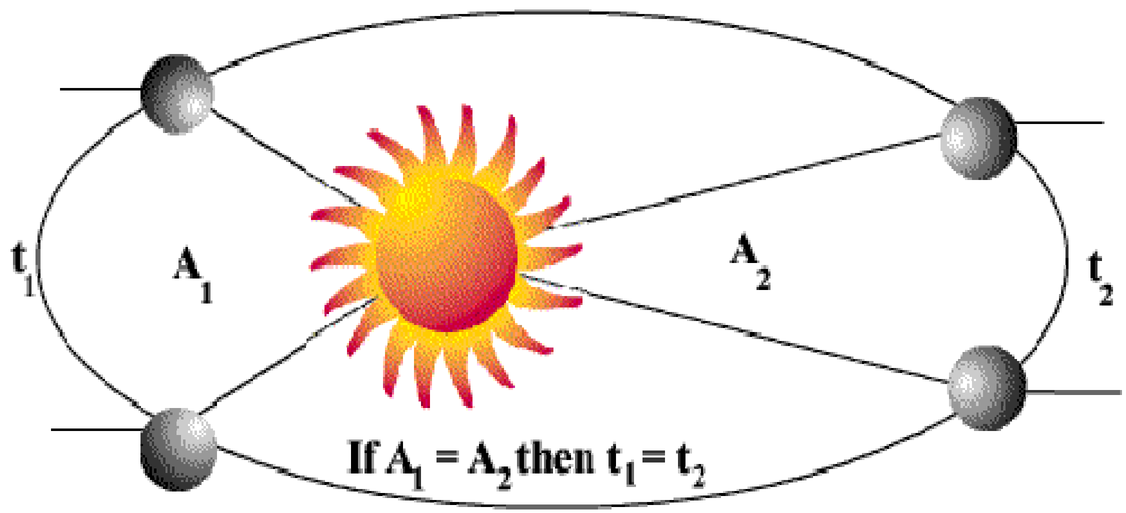
1. Law of Ellipses

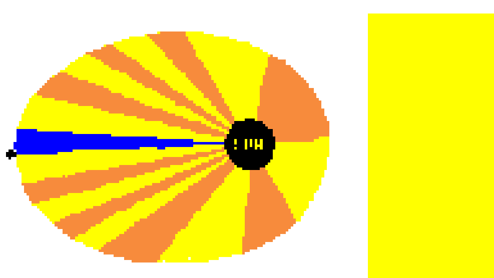
The orbits of planets are ellipses with the Sun at one focus



2. Law of Equal Areas

A line joining a planet to the Sun sweeps out equal areas in equal intervals of time

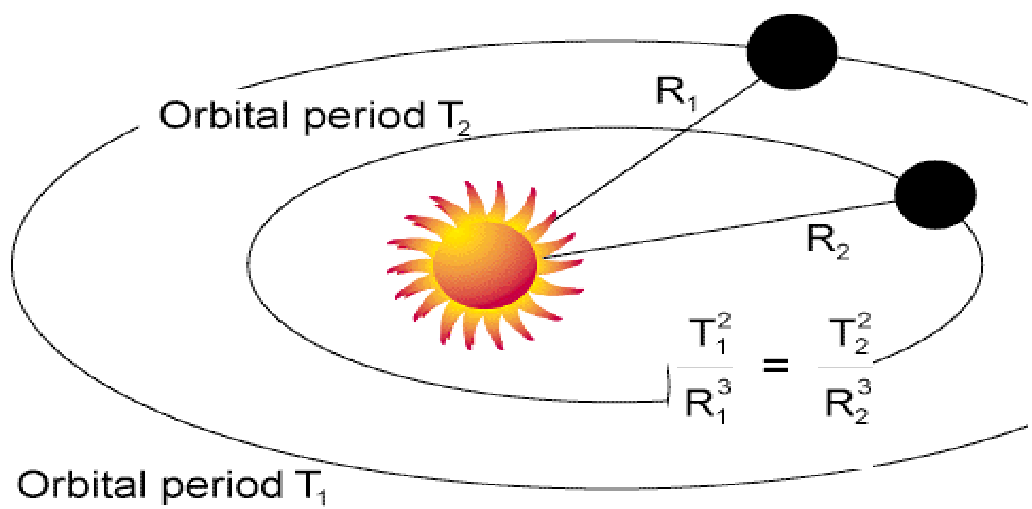




3. Harmonic Law

The ratio of the square of a planet's orbital period to the cube of its average orbital radius is constant

$$\frac{T_a^2}{T_b^2} = \frac{R_a^3}{R_b^3}$$



Kepler's Constant :*K*

$$\frac{R^3}{T^2} = K$$

Holds true so long as the bodies are orbiting the same focus,

i.e. it will hold true for the planets rotating around the Sun but not for the Moon.

Gravitational Potential Energy

Ep = mgh applies only when g is relatively constant and h is relatively small.

The work done on a body to raise it a distance d is still given by

$$W = F_g \times d$$

$$W = \frac{GMm}{R^2} \times d$$

$$W = \frac{GMm}{R}$$

Since $work_{done} = \Delta E_p$ from the Work energy theorem, and work was doneon the body so it must gain energy

But at some great distance from Earth Fg, acting on the body due to Earth = 0.

This suggests that

as distance→ towards ∞, Ep-→0,

and all bodies on Earth must have a negative Ep

$$\text{At} \infty \; E_p = 0$$

$$\therefore \Delta E_p = E_{p\infty} - E_{p\,\text{EARTH}}$$

$$\Delta E_p = 0 - E_{p\,\text{EARTH}}$$

but the work done is equal to the change in potential energy

$$\frac{GMm}{d} = 0 - E_p$$

$$\text{Then: } E_p = -\frac{GMm}{d}$$

Example:

Find the Ep of a 70.0 kg person on Earth with zero at infinity

Example solution

$$E_p = -\frac{GMm}{d}$$

$$= -\frac{(6.67\times10^{-11})(5.98\times10^{24})(70)}{6.38\times10^{6}}$$

$$= -4.4\times10^{9}\,\text{J}$$

Orbital Velocity

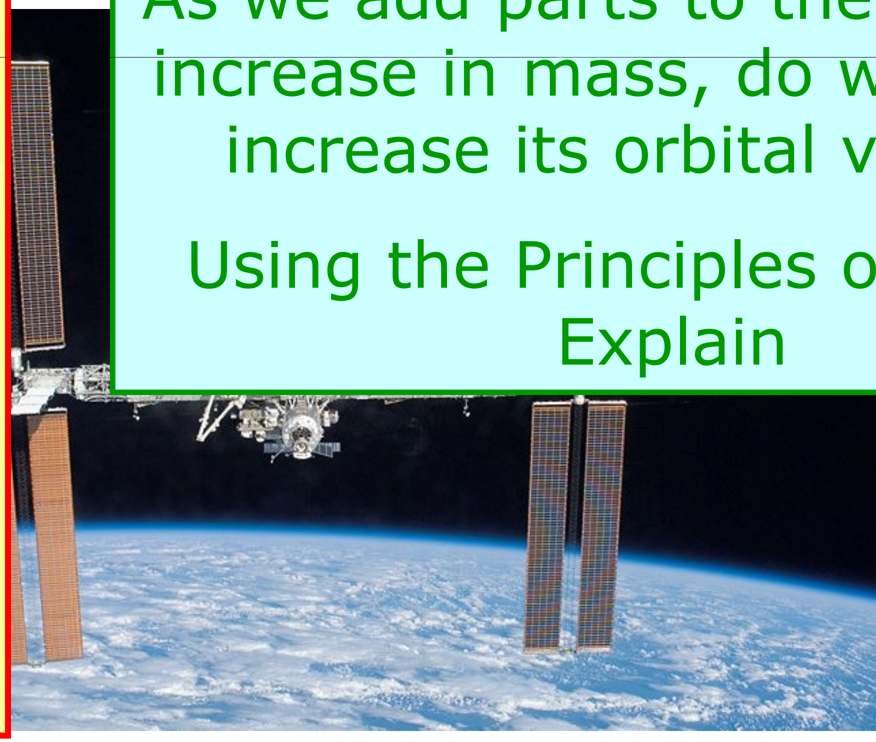
- The ISS (space station) has an average height of about 330 km above the Earth. How fast would it need to travel in order to maintain this orbit?

$$F_c = F_g$$

$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$

$$v^2 = \frac{GM}{R}$$

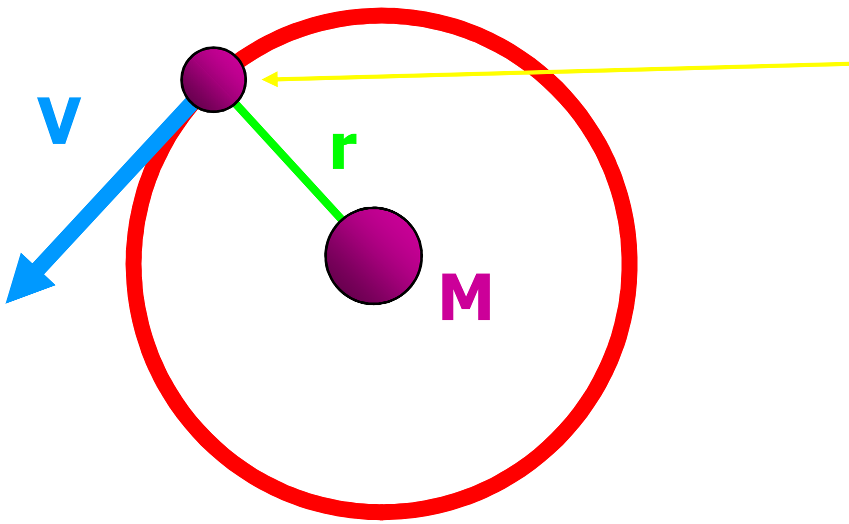
$$v_{orbital} = \sqrt{\frac{GM}{R}}$$



As we add parts to the ISS it will increase in mass, do we need to increase its orbital velocity?

Using the Principles of Physics Explain

So we can say that, the speed of an orbiting object depends only on the central mass*!

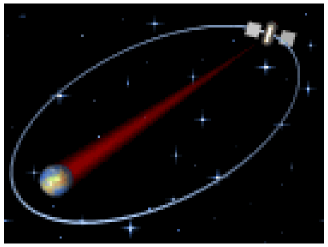


This object will always orbit with the same speed, no matter how heavy it is.

$$V_{rotation} = \sqrt{\frac{GM_{central}}{r}}$$

*and the separation from it

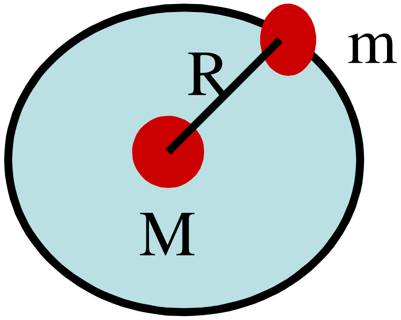
N.B. general solution is an ellipse not a circle - planets travel in ellipses around sun



Satellites

- Centripetal Force provided by Gravity

$$v = \sqrt{G\frac{M}{R}}$$

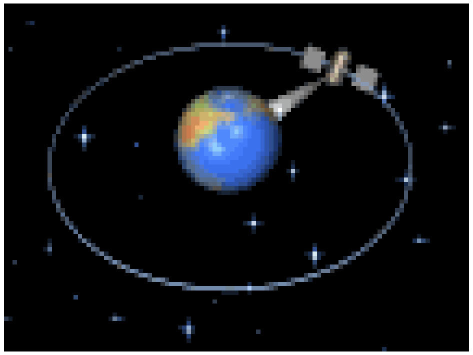


Distance in one revolution s = 2πR, in time period T,

$$T = 2\pi R / v = 2\pi R\sqrt{\frac{R}{GM}}$$

$T^2 \propto R^3$, Kepler's 3rd Law

- Special case of satellites – Geostationary orbit
  - Stay above same point on earth T = 24 hours



$$24 \times 60 \times 60 = 2\pi \frac{R^{3/2}}{\sqrt{GM_E}}$$

$$R = 42{,}000 km$$

Geosynchronous satellites orbit the Earth such that they always remain in a fixed spot above the Earth

- At what height above Earths surface must a satellite orbit so that its period is 24 hrs?

we need

$$\text{Fc} = \frac{\text{m}4\Pi^2 R}{T^2}$$

we have

$$\text{Fg} = \frac{\text{GMm}}{R^2}$$

$$\frac{\text{m}4\Pi^2 R}{T^2} = \frac{\text{GMm}}{R^2}$$

$$4\Pi^2 R^3 = GMT^2$$

$$R = \sqrt[3]{\frac{GMT^2}{4\Pi^2}}$$

$$R = 4.22\text{x}10^7 \text{m (from centre!)}$$

$$\text{h} = 42.2 \text{ x } 10^6 \text{m} - 6.38 \text{ x } 10^6 \text{m} = 3.58 \text{ x } 10^7 \text{m}$$

Escape Velocity, for unpowered projectiles!

Total mechanical energy: $\Delta E_K = \Delta E_P$

$$E = \frac{1}{2}mv^2 - G\frac{Mm}{r}$$

Conservation of mechanical energy:

$$\frac{1}{2}mv^2 = G\frac{Mm}{r} \Rightarrow v_{\mathrm{esc}} = \sqrt{2GM / r} = 11.2 \text{ km/s}$$

Minimal launch speed:

$$g = \frac{v^2}{r} \Rightarrow v_{\min} = \sqrt{Rg} = 7.9 \text{ km/s}$$